(DMSTT 01)

Assignment-1

M.Sc. DEGREE EXAMINATION, MARCH – 2023

First Year

STATISTICS

Probability and Distribution Theory

MAXIMUM MARKS: 30

Answer ALL Questions

Q1) a) Give the axiomatic definition of probability. If a set function assumes the value 1 at the empty set, show that it cannot be additive.

b) Define characteristic function. State its properties. State and prove inversion theorem.

Q2) a) Define expectation. State its properties. State and prove Holder's inequality. Hence obtain Minkowski inequality.

b) State and prove Borel – Cantelli lemma.

Q3) a) Define convergence in probability and almost sure convergence. Establish the inter relationships between convergence in distribution, convergence in probability and almost sure convergence.

b) State and prove Liapounov's form of central limit theorem.

Q4) a) State and prove Kolmogrov's strong law of large numbers.

b) Determine whether weak law of large numbers holds for the sequence of independent random variables:

$$P[X_n = n^{\alpha}) = \frac{1}{2n^2} = P(X_n = -n^2);\ P(X_n = 0) = 1 - \frac{1}{n^2},\ \left(\alpha < {}^{3}/_{2}\right).$$

Q5) a) Define multinomial distribution. Obtain its m.g.f. Hence find the m.g.f. of trinomial distribution.

b) Derive the distribution of compound Poisson.

(DMSTT 01)

Assignment-2

M.Sc. DEGREE EXAMINATION, MARCH – 2023

First Year

STATISTICS

Probability and Distribution Theory

MAXIMUM MARKS: 30

Answer ALL Questions

Q1) a) Find the m.g.f. and hence mean and variance of the negative binomial distribution.

b) Derive the distribution of compound binomial.

Q2) a) Define Weibull distribution. Obtain its characteristic function. Find the first two moments.

b) Define the log-normal distribution. State and prove its reproductive property.

Q3) a) Define logistic distribution and obtain its m.g.f.

b) Define Laplace distribution. Obtain its characteristic function. Hence obtain its mean and variance.

Q4) a) Derive the joint distribution of the j^{th} and k^{th} order statistics of $1 \leq j < k \leq n$.

b) Derive the distribution of non-central F and hence show that the central F is a particular case of it.

Q5) a) Define order statistics and obtain the distribution of i^{th} order statistic. Also obtain the same when the parent population is rectangular over $[0, \ 1]$.

b) Derive the distribution of non-central Chi-square.

(DMSTT 02)

Assignment-1
M.Sc. DEGREE EXAMINATION, MARCH – 2023
First Year
STATISTICS
Statistical Inference
MAXIMUM MARKS: 30
Answer ALL Questions

Q1) a) Define a sufficient statistic. If $X_1, X_2, \ldots, X_n$ is a random sample from $f(x : \theta) = \theta^x (1 - \theta)^{1-x}$, $x = 0, 1$, $0 < \theta < 1$.

Obtain a sufficient statistic for θ.

b) State and prove Cramer-Rao inequality.

Q2) a) Explain the concept of minimal sufficient statistics and describe how it is related to bounded completeness.

b) State and prove Lehmann-Scheffe theorem.

Q3) a) Explain the concepts of CAN and CAUN estimators. Explain the construction of CAN estimators based on moments.

b) Describe the maximum likelihood method of estimation. Find an ML estimator for θ in $f(x : \theta) = (\theta + 1)x^{\theta}, 0 \leq x \leq 1$.

Q4) a) Define efficiency and consistency. Prove that a function of a consistent estimator is also consistent.

b) Explain pivotal quantity method of finding confidence interval.

Q5) a) Distinguish between randomised and non-randomised tests. State and prove Neyman-Pearson lemma.

b) Find UMP test for testing $H_0 : \theta = \theta_0$ against one sided alternatives in $N(\theta, \sigma^2)$ when σ^2 is known.

(DMSTT 02)

Assignment-2

M.Sc. DEGREE EXAMINATION, MARCH – 2023

First Year

STATISTICS

Statistical Inference

MAXIMUM MARKS: 30

Answer ALL Questions

Q1) a) Explain likelihood ratio test. Obtain the asymptotic distribution of likelihood ratio test.

b) Let $X_1, X_2, \ldots. X_n$ be a random sample from a Poisson distribution with parameter λ. Derive the likelihood ratio tests for $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda > \lambda_0$ and $H_1 : \lambda < \lambda_0$. Show that they are identical with the corresponding UMP tests.

Q2) a) Explain Kolmogorov – Smirnov one sample and two sample tests.

b) Explain median test.

Q3) a) Explain Wilcoxon-Mann-Whitney test.

b) Explain sign test.

Q4) a) Explain SPR test procedure. Show that it terminates with probability one.

b) Derive the SPR test to test the binomial proportion and obtain its OC and ASN functions.

Q5) a) Determine the constants A and B in SPR test.

b) Derive the SPR test for testing the mean of a normal distribution with unit variance.

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(DMSTT 03)

Assignment-1

M.Sc. DEGREE EXAMINATION, MARCH – 2023

First Year

STATISTICS

Sampling Theory

MAXIMUM MARKS: 30

Answer ALL Questions

Q1) a) Explain the sources of sampling and non-sampling errors.

b) Discuss the main steps in conducting a sample survey.

Q2) a) Explain:

i) random number tables and their use.

ii) Census versus sample.

b) Explain the organisation and functions of CSO.

Q3) a) Explain simple random sampling with and without replacement. Obtain the variance of the estimated mean in SRS without replacement.

b) In the usual notation obtain the variance of P_{st}, the estimate appropriate to stratified random sampling for the population proportion.

Q4) a) Explain how the gain due to stratification is achieved.

b) In the usual notation show that $V(\bar{y}_n)_R \geq V(\bar{y}_{st})_P \geq V(\bar{y}_{st})_N$.

Q5) a) What is cluster sampling? Obtain an unbiased estimator of population mean based on clusters of equal size and derive an expression for the variance of this estimator.

b) Explain systematic sampling. Derive the variance of the estimated mean in systematic sampling.

(DMSTT 03)

Assignment-2
M.Sc. DEGREE EXAMINATION, MARCH – 2023
First Year
STATISTICS
Sampling Theory
MAXIMUM MARKS: 30
Answer ALL Questions

Q1) a) Determine the optimum cluster size so as to minimise the variance for a fixed cost.

b) Explain circular systematic sampling. Give any two of its applications.

Q2) a) Explain Lahiri's method of drawing a PPS sample. Obtain the variance of estimated population total in PPS sampling.

b) Explain multistage sampling. What are its advantages and disadvantages. Give any two of its applications.

Q3) a) Obtain an unbiased estimate of the population total in PPS sampling with replacement. Obtain an unbiased estimate of the variance of the estimated population total.

b) Derive the variance of the estimated mean in two-stage sampling with equal number of second stage units.

Q4) a) Define ratio estimates for population mean, population total and ratio and give examples for the use of these estimates.

b) Define separate and combined regression estimates in stratified random sampling and compare their variances. When do you use them in practice.

Q5) a) Show that the bias in the separate ratio estimator increases as the number of strata increases and the stratum sample size in each stratum is small.

b) Discuss the relative efficiency of ratio and regression estimates.

(DMSTT04)

Assignment-1

M.Sc. DEGREE EXAMINATION, MARCH – 2023

First Year

STATISTICS

Design of Experiments

MAXIMUM MARKS: 30

Answer ALL Questions

Q1) a) Define

i) Determinant

ii) Inverse of a matrix

iii) Orthogonal matrix.

Give examples.

b) Find the Characteristic roots and vectors of $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$.

Q2) a) State and prove Cauley-Hamilton theorem.

b) State Cochran's theorem for quadratic forms. Find the rank of $A = \begin{bmatrix} 1 & 6 & 0 \\ 1 & 2 & -4 \\ 0 & 1 & 1 \end{bmatrix}$.

Q3) a) Explain the linear model. State and prove a necessary and sufficient condition for the estimability of linear parametric functions.

b) State and prove Gauss – Markov theorem.

Q4) a) Explain the generalised linear model. Define best linear unbiased estimate.

b) State and prove Aitken's theorem.

Q5) a) Explain fixed, random and mixed effect models.

b) Explain two-way ANOVA with an equal number of observations.

(DMSTT04)

Assignment-2
M.Sc. DEGREE EXAMINATION, MARCH – 2023
First Year
STATISTICS
Design of Experiments
MAXIMUM MARKS: 30
Answer ALL Questions

Q1) a) Explain analysis of covariance of one-way classification.

b) Explain analysis of various of one-way classification with unequal number of observations.

Q2) a) Explain the principles of design of experiments.

b) Derive the statistic associated with testing the equality of K treatment effects in CRD.

Q3) a) Explain the analysis of LSD with a missing row or a missing column.

b) Determine the efficiency of RBD over CRD.

Q4) a) What are factorial experiments? Explain the analysis of 3^3 factorial experiment.

b) Define a BIBD. State and prove its parametric relations.

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Q5) a) Explain the analysis of 2^3 factorial experiment.

b) Discuss the intra block analysis of BIBD.

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