(DM21)

ASSIGNMENT - 1

M.Sc. DEGREE EXAMINATION, MARCH 2023

. Second Year

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Let f be a one-to-one mapping of one topological space onto another, and show that f is a homomorphism $\Leftrightarrow$ both f and f^{-1} are continuous.

   (b) Let X be a second countable space. Then any open base for X has a countable subclass which is also an open base.

2. (a) Prove that any continuous image of a compact space is compact.

   (b) State and prove generalised Heine-Borel theory.

3. (a) Prove that every compact metric space has the Bolzano-Weierstrass property.

   (b) Prove that a closed subspace of complete metric space is compact $\Leftrightarrow$ it is totally bounded.

4. (a) Prove that the product of any non-empty class of Hausdorff spaces is a Hausdorff space.

   (b) Prove that every compact Hausdorff space is normal.

5. (a) State an prove Urysohn's theorem.

   (b) Prove that a topological space is connected $\Leftrightarrow$ every non empty proper subset has a non-empty bundary.

(DM21)

ASSIGNMENT - 2

M.Sc. DEGREE EXAMINATION, MARCH 2023

. Second Year

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) If M is a closed linear subspace of a normed linear space N, and if T is a natural mapping of N onto N/M defined by $T(x) = x + M$, show that T is a continuous linear transformation for which $\| T \| \leq 1$.

   (b) State and prove the Hahn-Banach theorem.

2. (a) Show that a linear subspace of a normed linear space is closed it is weakly closed.

   (b) Explain the open mapping theorem.

3. (a) Prove that if B is reflexive Banach space, then its closed unit sphere 'S' is weakly compact.

   (b) State and prove, the Uniform Boundedness theorem.

4. (a) Prove that A closed convex subset 'C' of a Hilbert space H contains a unique vector of smallest noun.

   (b) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then the linear subspace $M + N$ is also closed

5. (a) State and prove Bessel's Inequality.

   (b) Show that the adjoint operation is one-to-one as a mapping of $B(H)_f$ into itself.

(DM22)

ASSIGNMENT - 1

M.Sc. DEGREE EXAMINATION, MARCH 2023

Second Year

Mathematics

MEASURE AND INTEGRATION

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Let $\langle f_n \rangle$ be a sequence of continuous functions defined on R. Show that the set C of points where this sequence converges in on Fab.

   (b) Show that $\inf E < \sup E$ if and only if $E \neq \phi$

2. (a) Prove that if E_1 and E_2 are measurable so is $E_1 \cup E_2$.

   (b) Let $\langle E_i \rangle$ be a sequence of disjoint measurable sets and A is any set then $m^*(A \cap \bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} m^*(A \cap E_i)$

3. (a) Prove that the product of two measurable extended real valued functions is measurable.

   (b) State and prove Egoroff's theorem.

4. (a) State and prove Bounded convergence theorem.

   (b) Let f be a non negative measurable function. Show that $\int f = 0 \Rightarrow f = 0$ $a.e.$

5. (a) Let $\langle f_n \rangle$ be a sequence of integrable functions such that $\int |f - f_n| \to 0$ if and only if $\int |f_n| \to \int |f|$

   (b) Explain convergence in measure.

(DM22)

ASSIGNMENT - 2

M.Sc. DEGREE EXAMINATION, MARCH 2023

Second Year

Mathematics

MEASURE AND INTEGRATION

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) A function f is of bounded variation on $[a, b]$ if and only if f is the difference of two monotone real valued functions on $[a, b]$.

   (b) Let f be an integrable function on $[a, b]$ and suppose that

   $$F(x) = F(a) + \int_a^x f(t)\, dt$$

   then $F'(x) = f(x)$ for almost all x in $[a, b]$

2. (a) State and prove Minkowski inequality for $0 < p < 1$.

   (b) Prove that every convergence sequence is a Cauchy sequence.

3. (a) Show that μ is σ-finite if and only if all but a countable number of the μ_2 are zero and the remainder are σ-finite.

   (b) State and prove Monotone convergence theorem.

4. (a) Show that if E is any measurable set, then

   $-vE \le vE \le v^t E$ and $|vE| \le |v|(E)$

   (b) Prove the uniquencess ascertain in the Lebsegue decompositon.

5. (a) Show that an outer measure μ^* is regular if and only if it is induced by a measure of an algebra.

   (b) Assume that $\langle E_i \rangle$ is a sequence of disjoint measurable sets and $E = \cup E_i$. Then for any set A we have $\mu^*(A \cap E) = \sum \mu^*(A \cap E_i)$

(DM23)

ASSIGNMENT - 1

M.Sc. DEGREE EXAMINATION, MARCH 2023

Second Year

Mathematics

ANALYTICAL NUMBER THEORY AND
GRAPH THEORY

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) If $n \geq 1$, then prove that $\sum_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$

   (b) Use Euler's formula to deduce $\sum_{n \leq x} \frac{\log x}{n} = \frac{1}{2}\log^2 x + A + O\left(\frac{\log x}{x}\right)$, where A is constant and $x \geq 2$.

2. (a) If $n \geq 2$, prove that $\sum_{n \leq x} \frac{1}{\phi(n)} = O(\log x)$.

   (b) The set of all lattice points visible from the origin has density $6/\pi^2$

3. (a) For $n > 1$, the n^{th} prime p, satisfies the inequality.
   $\frac{1}{6} n \log n < p_n < 12\left(n \log n + n \log \frac{12}{e}\right)$

   (b) Prove that, there is a constant A, such that
   $\sum_{p \leq x} \frac{1}{p} = \log \log x + A + O\left(\frac{1}{\log x}\right) \; \forall \; x \geq 2$.

4. (a) Prove that for every $n > 1$ there exist n consecutive composite numbers.

   (b) If $x \geq 2$, Let $Li(x) = \int_2^x \frac{dt}{\log t}$. Then prove that $Li(n) = \frac{x}{\log x} + \int_2^x \frac{dt}{\log^2 t} - \frac{2}{\log 2}$

5. (a) Prove that a simple graph with 'n' vertices must be connected if it has move that $(n-1)(n-2)/2$ Edges.

   (b) In a graph G, Let P_1 and P_2 are two different paths between two given vertices, prove that $P_1 \oplus P_2$ is a circuit (or) a set of circuits in G.

(DM23)

ASSIGNMENT - 2

M.Sc. DEGREE EXAMINATION, MARCH 2023

Second Year

Mathematics

ANALYTICAL NUMBER THEORY AND
GRAPH THEORY

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Explain Euler's graph with an example.

   (b) Prove that a connected graph G remains connected after removing an edge e_i from G if and only if e_i is in same circuit in G.

2. (a) Prove that a tree with 'n' vertices has exactly $(n-1)$ Edges.

   (b) Show that a Hamiltonian path is a spanning tree.

3. (a) Prove that every circuit has an even number of edges in common with any cut-set.

   (b) Explain cut point and cut Edge with examples.

4. (a) Prove that a connected planar graph with n vertices has $e-n+2$ regions.

   (b) A complete bipartite graph Km, n ¸is planar if and only if $m \leq 2$ or $n \leq 2$.

5. (a) Prove that the ring sum of two circuits in a graph G is either a circuit or an edge-disjoint union of circuits.

   (b) Explain Basis vectors of a Graph.

(DM24)

ASSIGNMENT - 1

M.Sc. DEGREE EXAMINATION, MARCH 2023

Second Year

Mathematics

RINGS AND MODULES

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Show that a group may equivalently be defined as a system (S, 1, /), where / is a binary operation satisfying the identifies.

   $a/1=a$, $a/a=1$, $(a/c)(b/c)=a/b$

   (b) Prove that $Sup\,T=\inf T^{V}$, where

   $T^{V}=\{s\in S/\forall t\in T^{t}\leq s\}$

2. (a) If θ is a homomorphic relation between R and S and T is a subring of S, show that $\theta T=\{r\in R/\exists t\in T^{r}\theta t\}$ is a subring of R.

   (b) Prove that if a ring is a sum of ideals, then it is a finite sum.

3. (a) Show that any Artivian or Noetherian module can be written as a direct sum of indecomposable modules.

   (b) Prove that if $\phi\in Hom_{R}(A,B)$ then $\phi A\cong A/\phi^{-1}O$.

4. (a) Determine all prime and maximal ideals as well as both radicals of Z(n), the ring of integers Modulo '*n*'.

   (b) Prove that every ring is a sub direct product of sub directly irreducible rings.

5. (a) If M is a maximal ideal in the ring R and n is any positive integer, show that R/M^{n} has a unique prime ideal.

   (b) Prove that, if R is a commutative ring, then the system $(F,0,1,-,+,\bullet)/\theta=Q(R)$ is also a commutative ring. It extends R and will be called its complete ring of quotients.

(DM24)

ASSIGNMENT - 2

M.Sc. DEGREE EXAMINATION, MARCH 2023

Second Year

Mathematics

RINGS AND MODULES

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Show that R is a prime ring if and only if $1 \neq 0$ and $AB \neq 0$ for any two non zero right ideals A and B of R.

   (b) State and prove Chinese Remainder theorem.

2. (a) If R is a commutative ring, show that every dense ideal is large, and that conversely every large ideal is dense, if and only if R is semi primitive.

   (b) Show that R is completely reducible if and only if no maximal right ideal is large.

3. (a) If D and D' are division rings and $D_n \cong D'n'$ show that $D \cong D'$ and $n = n'$.

   (b) Show that every factor ring of a right Noetherian (Artivian) ring is right Noetherian (Artivian).

4. (a) Prove that every free module is projective.

   (b) Prove that if R^F is a free module then F_R^* is injective.

5. (a) Show that every R-module is injective if and only if R is completely reducible.

   (b) Show that a ring is right hereditary if and only if every submodule of a projective module is projective.