(DM01)

ASSIGNMENT - 1

M.Sc. DEGREE EXAMINATION, MARCH 2023

First Year

Mathematics

ALGEBRA

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) If N, M are normal subgroups of G prove that $NM / M \approx N / N \cap M$.

   (b) Let G be a group, H is a subgroup of G, T is an automorphism of G, Let $(H)T = \{ hT / h \in H \}$. Prove that $(H)T$ is a subgroup of G.

2. (a) State and prove Cayley theorem.

   (b) If H is a subgroup of G, show that for every $g \in G$, gHg^{-1} is a subgroup of G.

3. (a) Express as a product of disjoint cycles
   (1, 2, 3) (4, 5) (1, 6, 7, 8, 9) (1, 5)

   (b) Find the number of conjugates of (1, 2) (3, 4) is $S_n, n \geq 4$.

4. (a) If D is an integral Domain and D is of finite characteristics prove that the characteristic of D is a prime number.

   (b) If F is a field, prove that its only ideals are (O) and F itself.

5. (a) Prove that the mapping $\phi : D \to F$ defined by $\phi(a) = [a, 1]$ is an isomorphism of D into itself.

   (b) If $a + bi$ is not a unit of $J[i]$, prove that $a^2 + b^2 > 1$.

(DM01)

ASSIGNMENT - 2

M.Sc. DEGREE EXAMINATION, MARCH 2023

First Year

Mathematics

ALGEBRA

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Suppose that F is a field having a finite number of elements q.

   (i) Prove that $q = p^n$ for same integer 'n'.

   (ii) If $a \in F$, prove that $a^q = a$.

   (b) If p is a prime number, prove that the splitting field over F, the field of rational numbers of the polynomial $x^p - 1$ is degree $p-1$.

2. (a) Prove that if α, β are constructable, then so are $\alpha \pm \beta, \alpha\beta$ and α/β are constructable.

   (b) Prove that $(f(x)+g(x))' = f'(x)+g'(x)$ and that $(\alpha f(x))' = \alpha f'(x)$ for $f(x),\ g(x) \in F(x),\ \ \alpha \in F$

3. (a) K is a normal extension of F if and only if K is splitting field of some polynomial over F.

   (b) Prove that S_4 is a soluable group.

4. (a) Show that $(q), q \neq 0, 1$ is primary in I if and only if $q = p^e,\ p$ is prime.

   (b) Explain complemented distributive lattices.

5. (a) If $\{L, \leq\}$ is a lattice, then for any $a, b, c \in L$,

   $a \leq c \Leftrightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c$

   (b) In any Boolean Algebra, show that

   (i) $(x+y)(x'+z) = xz + x'y + yz = xz + x'y$

   (ii) $(xy'z' + xy'z + xyz + xyz')(x+y) = x$

(DM02)

ASSIGNMENT - 1

M.Sc. DEGREE EXAMINATION, MARCH 2023

First Year

Mathematics

ANALYSIS

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Every infinite subset of a countable set A is countable.

   (b) Prove that, it P is a limit point of a set E, then every neighbourhood of P contain infinitely main points of E.

2. (a) Let P is a nonempty perfect set in R^k. Then P is uncountable

   (b) If a set E in R^k has one of the following three properties, then it has other two.

   (i) E is closed and bounded

   (ii) E is compact

   (iii) Every infinite subset of E has a limit point in E.

3. (a) Prove that every bounded sequence in R^k contains a convergent subsequence.

   (b) Prove that $\underset{n \to \infty}{Lt} \left(1 + \frac{1}{n}\right)^n = e$

4. (a) Suppose f is a continuous mapping of a compact metric space 'X' into a metric space Y. Then $f(x)$ is compact.

   (b) Let f be monotonically increasing n (a, b). Then $f(n+)$ and $f(n-)$ at every point of $x \in (a, b)$ and

   $\sup_{a < t < x} f(t) = f(n-) \le f(n) \le f(n+) = \text{int } f(t)$

   $a < t < x$

5. (a) If f is monotonic on $[a, b]$ and if x is continuous on $[a, b]$ then $f \in R(\alpha)$.

   (b) If P^α is an element of P, then

   (i) $L(P, f, \alpha) \le L(P^\alpha, f, \alpha)$

   (ii) $L(P^\alpha, f, \alpha) \le U(p, f, \alpha)$

(DM02)

ASSIGNMENT - 2

M.Sc. DEGREE EXAMINATION, MARCH 2023

First Year

Mathematics

ANALYSIS

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1.(a) If $f \in \mathbb{R}(\alpha)$ and $g \in \mathbb{R}(\alpha)$ on $[a, b]$ then

(i) $f\,g \in \mathbb{R}(\alpha)$

(ii) $|f| \in \mathbb{R}(\alpha)$ and $\left|\int_a^b f\,d\alpha\right| \leq \int_a^b |f|\,d\alpha$

(b) State and prove Fundamental theorem of calculus.

2.(a) State and prove Weierstress approximation theorem.

(b) Test for uniform convergence of the series $\sum \frac{\cos nx}{z^n}$

3.(a) Examine the series for uniform convergence

$$\sum_{n=1}^{\infty} \frac{1}{n^p + n^q x^2}$$

(b) If K is compact, if $f_n \in \mathscr{C}(k)$ for $n = 0, 1, 2, \ldots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on k, then

(i) $\{f_n\}$ is uniformly bounded on K,

(ii) $\{f_n\}$ is continuous a uniformly convergent subsequence.

4. (a) If $\int_A f\,d\mu = 0$ for every measurable subset A of a measurable set E, then $f(n) = 0$ almost everywhere on E.

(b) If $f, g \in L(\mu)$ on X, define the distance between f and g by

Prove that $L(\mu)$ is complete metric space.

5. (a) State and prove Fatau's theorem.

(b) State and prove Lebegue's dominated convergence theorem.

(DM03)

ASSIGNMENT - 1

M.Sc. DEGREE EXAMINATION, MARCH 2023

First Year

Mathematics

COMPLEX ANALYSIS AND SPE. FUNCTIONS AND PARTIAL DIF. EQU.

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Show that $(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$

   (b) Prove that $\int_{-1}^{1} [P_2(x)]^2 dx = \frac{2}{5}$

2. (a) Express $x^4 + 3x^3 - x^2 + 5x - 2$ interfy of legendre polyaxials.

   (b) Prove that $P_{2n}(0) = \frac{(-1)^n (2n)!}{2^{2n} (n!)^2}$

3. (a) Prove that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

   (b) Prove that $J_0"(x) = \frac{1}{2}\left[J_2(x) - J_0(x)\right]$

4. (a) Form the partial differential equation by Eliminating arbitrary function $Z = x^2 f(x) + y^2 g(x)$

   (b) Solve $(mz - ny)p + (nx - lz)q = (ly - mx)$

5. (a) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{2\,\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2$

   (b) Solve $t - r \sec ty = 2q \tan y$

(DM03)

ASSIGNMENT - 2

M.Sc. DEGREE EXAMINATION, MARCH 2023

First Year Mathematics

COMPLEX ANALYSIS AND SPE. FUNCTIONS AND PARTIAL DIF. EQU.

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Determine the analytic function whose Real part is $e^{-x}[x \sin y - y \cos y]$.

   (b) Show that $f(z) = \sqrt{|xy|}$ is not analytic at the origin even-though $C-R$ equations satisfied at the origin.

2. (a) State and prove Cauchy's theorem.

   (b) Evaluate $f(2)$ and $f(3)$ where

   $$f(a) = \int_C \frac{2z^2 - z^2}{z-a}\, dz \text{ and } C \text{ is a circle } |z| = 2.5$$

3. (a) State and prove open mapping theorem.

   (b) Expand $f(z) = \dfrac{7z^2 - 9z - 18}{z^3 - 9z}$ in the region

   (i) $|z| > 3$, (ii) $0 < |z-3| < 3$.

4. (a) State and prove Residue theorem.

   (b) Evaluate $\oint_C \dfrac{z-3}{z^2 - 2z + 5}\, dz$, where C is the circle.

   (i) $|z| = 1$

   (ii) $|z + 1 - i| = 2$

   (iii) $|z + 1 + i| = 2$

5. (a) Show that

   $$\int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{1 - 2a\cos\theta + a^2} = \frac{2\pi a^2}{1-a^2}, (a^2 < 1)$$

   (b) Show that $\displaystyle\int_0^{+\infty} \frac{1}{x^6+1}\, dx = \frac{\pi}{3}$

(DM04)

ASSIGNMENT - 1

M.Sc. DEGREE EXAMINATION, MARCH 2023

First Year

Mathematics

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Find two linearly independent solutions of the equation.

$$(3x-1)^2 \frac{d^2y}{dx^2} + (9x-3)\frac{dy}{dx} - 9y = 0 \text{ for } x > \frac{1}{3}.$$

(b) Verify that the function ϕ_1 satisfies the equation, and find second independent solution.

$$x^2y'' - 7xy' + 15y = 0,\ \phi_1(x) = x^3,\ (x > 0).$$

2. (a) Find all solutions of the equation

$$y'' - \frac{2}{x^2}y = x\ ,\ 0 < x < \infty\ .$$

(b) Find two linearly independent power series solutions of the equation

$$y'' + 3x^2y' - xy = 0\ .$$

3. (a) Find all real-values solutions of $y' = x^2y^2 - 4x^2$.

(b) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation

$$y' = x^2 + y^2\ ,\ y(0) = 0\ .$$

4. (a) By computing appropriate Lipschitz constants, show that the function $f(x, y)$ satisfy Lipschitz conditions

$$f(x, y) = x^2\cos^2 y + y\sin^2 x \text{ on } S, \text{ for } |x| \le 1, |y| \le \infty$$

(b) Let f be a real valued continuous function on the strip $S: |x| \le a, |y| \le \infty, a > 0$ and f satisfies Lipschitz condition on 'S'. Show that the solution of initial value problem $y'' + \lambda^2 y = f(x, y)$, $y(0) = 0$, $y'(0) = 1$ is unique $(\lambda > 0)$.

5. (a) Find a solution ϕ of $y'' = \frac{-1}{2y^2}$ satisfying $\phi(0) = 1$, $\phi'(0) = -1$.

(b) Suppose

$Y = (8 + i,\ 3i,\ -2)$,

$Z = (i, -1, -2)$, $W = (2 + i, 0, 1)$ then compute

(i) $y - z$

(ii) Show for some number S

$$W = z + s(y - z)\ .$$

(DM04)

ASSIGNMENT - 2

M.Sc. DEGREE EXAMINATION, MARCH 2023

First Year

Mathematics

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

MAXIMUM : 30 MARKS

ANSWER ALL QUESTIONS

1. (a) Find a solution ϕ of the system $y_1' = y_1 + y_2$, $y_2' = y_1 + y_2 + e^{2x}$ satisfying $\phi(0) = (0, 0)$.

   (b) Consider the system

   $y_1' = 3y_1 + xy_3$, $y_2' = y_2 + \delta^3 y_3$,

   $y_3' = 2xy_1 - y_2 + e^x y_3$

   Show that every initial value problem for this system has a unique solution which exists for all real 'x'.

2. (a) Explain the Riccati equation.

   (b) Explain Green's functions.

3. (a) State and prove Abel's formula.

   (b) Explain the Strun separation theorem.

4. Explain the Bucher Osgood theorem.

5. (a) State and prove Liapunov's inequality.

   (b) Explain oscillations on a half axis two transformations.