

(DM01)

ASSIGNMENT-1

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2018

First Year

MATHEMATICS

Algebra

MAXIMUM MARKS:30

Answer ALL Questions

- Q1)** a) State and prove the Cayley's theorem.
- b) If ϕ is a homomorphism of a group G into \bar{G} , then show that
- i) $\phi(e) = \bar{e}$, the unit element of \bar{G} .
- ii) $\phi(x^{-1}) = [\phi(x)]^{-1}$ for all $x \in G$.
- Q2)** a) Suppose G is a group and that G is the internal direct product of N_1, N_2, \dots, N_n . If $T = N_1 \times N_2 \times \dots \times N_n$ then prove that G and T are isomorphic.
- b) If p is a prime number and $p^\alpha \mid O(G)$, then prove that G has a subgroup of order p^α .
- Q3)** a) State and prove the Cauchy's theorem for abelian groups.
- b) Define a permutation. Resolve the permutation
- $$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 2 & 5 & 6 & 4 & 8 & 7 \end{pmatrix}$$
- into disjoint cycles. Given $x = (1, 2)(3, 4)$, $y = (5, 6)(1, 3)$, find a permutation a such that $a^{-1}xa = y$.
- Q4)** a) Prove that every integral domain is a field.
- b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.
- Q5)** a) State and prove the Fermat's theorem.
- b) State the Eisenstein criterion, prove that the polynomial $1 + x + \dots + x^{p-1}$, where p is prime number, is irreducible over the field of rational numbers.

(DM01)

ASSIGNMENT-2

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2018

First Year

MATHEMATICS

Algebra

MAXIMUM MARKS:30

Answer ALL Questions

- Q1)** a) If L is a finite extension of K and K is a finite extension of F , then prove that L is a finite extension of F and that $[L : F] = [L : K][K : F]$.
- b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.
- Q2)** a) Prove that it is impossible, by straight edge and compass alone, to trisect 60° .
- b) Prove that K is a normal extension of a field F if and only if K is the splitting field of some polynomial over F .
- Q3)** a) Show that a general polynomial of degree n , $n \geq 5$ is not solvable by radicals.
- b) i) Prove that the fixed field of G is a sub field of K .
- ii) If K is a finite extension of F , then $G(K, F)$ is a finite group and show that $O(G(K, F)) \leq [K : F]$.
- Q4)** a) State and prove the Schreier's theorem.
- b) Derive the dimensionality equation $d(a \vee b) = d(a) + d(b) - d(a \wedge b)$ for modular lattice.
- Q5)** a) Prove that every distributive lattice with more than one element can be represented as a sub direct union of two element chains.
- b) Define a Boolean algebra and a Boolean ring. Show that a Boolean ring can be converted into a Boolean algebra.

(DM02)

ASSIGNMENT-1
M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2018
First Year
MATHEMATICS
Analysis
MAXIMUM MARKS:30
Answer ALL Questions

Q1) a) Let $\{E_n\}$, $n = 1, 2, 3, \dots$ be a sequence of countable sets and put $S = \bigcup_{n=1}^{\infty} E_n$.
Then show that S is countable.

b) Prove that closed subsets of compact sets are compact.

Q2) a) If a set E in \mathbb{R}^k has one of the following three properties, then show that it has the other two:

i) E is closed and bounded

ii) E is compact

iii) Every infinite subset of E has a limit point in E.

b) Suppose $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

Q3) a) Suppose $\{S_n\}$ is monotonic. Then prove that $\{S_n\}$ converges if and only if it is bounded.

b) Show that the product of two convergent series need not converge and may actually diverge.

Q4) a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that $f(X)$ is compact.

b) Define f on \mathbb{R}^1 by $f(x) = \begin{cases} 0, & \text{if } x \text{ is irrational} \\ \frac{1}{n}, & \text{if } x \text{ is rational} \end{cases}$

Show that f is continuous at every irrational point and it has a simple discontinuity at rational points.

- Q5)** a) State and prove a necessary and sufficient condition for a bounded function f to be R – S integrable on $[a, b]$.
- b) Suppose that $f \in \square(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi f(x)$ on $[a, b]$. Then show that $h \in \square(\alpha)$ on $[a, b]$.

(DM02)

ASSIGNMENT-2

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2018

First Year

MATHEMATICS

Analysis

MAXIMUM MARKS:30

Answer ALL Questions

Q1) a) Let f maps $[a, b]$ into \mathbb{R}^k and suppose that $f \in \square(\alpha)$ for some monotonically increasing function α on $[a, b]$. Then show that $|f| \in R(\alpha)$ and

$$\left| \int_a^b f d\alpha \right| \leq \int_a^b |f| d\alpha.$$

b) State and prove the fundamental theorem of integral calculus.

Q2) a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

b) If K is compact, $f_n \in \square(K)$ for $n = 1, 2, 3, \dots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K then prove that

i) f_n is uniformly bounded on K .

ii) $\{f_n\}$ contains a uniformly convergent subsequence.

Q8) State and prove the Weirstrass approximation theorem.

Q9) a) State and prove the Lebesgue's monotone convergence theorem.

b) If $\{f_n\}$ is a sequence of measurable functions then prove that the set of points x at which $\{f_n(x)\}$ converges is measurable.

Q10) a) Suppose $f = f_1 + f_2$ where $f_i \in \alpha(u)$ on E ($i = 1, 2$). Then show that $f \in \alpha(u)$ on E and $\int_E f d\mu = \int_E f_1 d\mu + \int_E f_2 d\mu$.

b) If $f \in \mathcal{R}$ on $[a, b]$, then show that $f \in L$ on $[a, b]$ and that

$$\int_a^b f dx = R \int_a^b f dx$$

(DM03)

ASSIGNMENT-1

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2018

First Year

MATHEMATICS

Complex Analysis & Spe. Functions & Partial Dif. Equ.

MAXIMUM MARKS:30

Answer ALL Questions

- Q1)** a) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$.
- b) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.
- Q2)** a) Prove that $\frac{1 - z^2}{(1 - 2xz + z^2)^{3/2}} = \sum_{n=0}^{\infty} (2n + 1) z^n P_n$.
- b) Find the solution of the Bessel equation of order n and of the first kind, n being a non negative constant.
- Q3)** a) Prove that the necessary condition for the integrability of the total differential equation $\bar{A} \cdot d\bar{r} = P dx + Q dy + R dz = 0$ is $\bar{A} \cdot \text{curl } \bar{A} = 0$.
- b) Solve $(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0$.
- Q4)** a) Solve $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$.
- b) Solve $y^2 r - 2y s + t = p + 6y$ using Monge's method.
- Q5)** a) Solve the partial differential equation $(x^2 + y^2 + yz)p + (x^2 + y^2 - xz)q = z(x + y)$, with the usual notation.
- b) Find the general solution of the partial differential equation $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^x$.

(DM03)

ASSIGNMENT-2

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2018

First Year

MATHEMATICS

Complex Analysis & Spe. Functions & Partial Dif. Equ.

MAXIMUM MARKS:30

Answer ALL Questions

Q1) a) Let u and v be real valued functions defined on a region G and suppose that u and v possess continuous partial derivatives. Then prove that $f: G \rightarrow \mathbb{C}$ defined by $f(z) = u(z) + i v(z)$ is analytic if and only if u and v satisfy the Cauchy – Riemann equations.

b) If $\sum a_n (z - a)^n$ is a given power series with radius of convergence R , then show that $R = \lim \left| \frac{a_n}{a_{n+1}} \right|$, if this limit exist. Find the radius of convergence of

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}.$$

Q2) a) Let R be a closed polygon $[1 - i, 1 + i, -1 + i, -1, -i, 1 - i]$. Then evaluate $\int_R \frac{1}{z} dz$.

b) State and prove the open mapping theorem.

Q3) a) Define Mobius transformation. Let z_1, z_2, z_3, z_4 be four distinct points in \mathbb{C}_{∞} . Then show that (z_1, z_2, z_3, z_4) is a real number iff all four points lie on a circle.

b) If $r: [a, b] \rightarrow \mathbb{C}$ is a piecewise smooth curve then prove that r is of bounded variation and $\nu(r) = \int_a^b |r'(t)| dt$.

Q4) a) State and prove the Laurent series development.

b) Find the Laurent series expansion of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ about $z = 0$ in the region $2 < |z| < 3$.

Q5) a) Discuss the nature and classification of singularities of a function $f(z)$. Find the nature and location of singularities of

i) $f(z) = e^z / (z-1)^4$

ii) $z^2 - 1 / (z-1)^3$

b) State the residue theorem. Using the residue theory evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx$.

EEE

(DM04)

ASSIGNMENT-1

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2018

First Year

MATHEMATICS

Theory of Ordinary Differential Equations

MAXIMUM MARKS:30

Answer ALL Questions

- Q1)** a) If $\phi_1, \phi_2, \dots, \phi_n$ are n solutions of $L(y) = 0$ on an interval I , prove that they are linearly independent there if and only if $w(\phi_1, \phi_2, \dots, \phi_n)(x) \neq 0$ for all x in I , where $L(y) = y^{(n)} + a_1(x)y^{n-1} + \dots + a_n(x)y = 0$.
- b) If one solution of $y'' - \frac{2}{x^2}y = 0, 0 < x < \infty$ is $\phi_1(x) = x^2$, find the general solution of the equation $y'' - \frac{2}{x^2}y = x$.
- Q2)** a) Find two linearly independent power series solutions of the equation $y'' - xy = 0$.
- b) Compute the solution of $y''' - xy = 0$ which satisfies $\phi(0) = 1, \phi'(0) = 0, \phi''(0) = 0$.
- Q3)** a) Let M, N be real valued functions having continuous first partial derivatives on some rectangle $R: |x - x_0| \leq a, |y - y_0| \leq b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R .
- b) Find an integrating factor of the equation $\cos x \cdot \cos y \, dx - 2 \sin x \cdot \sin y \, dy = 0$ and solve it.

Q4) a) Show that the function ϕ is a solution of the I.V.P. $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation

$$y = y_0 + \int_{x_0}^x f(t, y) dt \text{ on I.}$$

b) Let $f(x, y) = \frac{\cos y}{1 - x^2}$; ($|x| < 1$), show that f satisfies a Lipschitz condition on every strip $s_a : |x| \leq a$, where $0 < a < 1$. Show that every initial value problem $y' = f(x, y)$, $y(0) = y_0$, ($|y_0| < \infty$) has a solution which exists for $|x| < 1$.

(DM04)

ASSIGNMENT-2

M.Sc. (Previous) DEGREE EXAMINATION, MAY – 2018

First Year

MATHEMATICS

Theory of Ordinary Differential Equations

MAXIMUM MARKS:30

Answer ALL Questions

- Q1)** a) Find the solution of $y'' = -\frac{1}{2y^2}$ satisfying $\phi(0) = 1, \phi'(0) = -1$.
- b) Find a solution ϕ of the system $y_1' = y_1, y_2' = y_1 + y_2$ which satisfies $\phi(0) = (1, 2)$.
- Q2)** a) State and prove the local existence theorem for the existence of solution to the system: $Y' = f(x, Y), Y(x_0) = Y_0$.
- b) Show that all real valued solutions of the equation $y'' + \sin y = b(x)$, where b is continuous for $-\infty < x < \infty$, exist for all real x .
- Q3)** a) Find the general solution of the Riccate equation.
- b) Find the functions $Z(x), K(x)$ and $m(x)$ such that
- $$z(x)[x^2 y'' - 2xy' + 2y] = \frac{d}{dx}[k(x)y' + m(x, y)].$$
- Q4)** a) Show that the Green's function for $L(x) = x'' = 0, x(0) + x(1) = 0, x'(0) + x'(1) = 0$ is $G(t, s) = 1 - s$ if $t \leq s$ and $G(t, s) = 1 - t$ if $t \geq s$.
- b) Find the general solution of $y'' - 4y' + 3y = x, (-\infty < x < \infty)$ by computing the particular solution using Green's theorem.
- Q5)** State and prove the sturm separation theorem.
- Q6)** a) Express the differential equation $x^2 y'' + x y' + (x^2 - n^2)y = 0$ in self-adjoint form.
- b) State and prove the Gronwall's inequality.