(DMSTT01)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Statistics

PROBABILITY AND DISTRIBUTION THEORY
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Define the probability distribution of a random variable. State and prove its properties.

   (b) Define characteristic function and state its properties. Derive inversion formula.

2. (a) Obtain the mean and variance of the distribution function :

$$F(x) = 0, x < 0$$
$$= p + (1-p)\left(1 - e^{-\lambda x}\right), 0 < x < T$$
$$= 1 \qquad\qquad , x \geq T$$

   (b) State and prove Borel - Cantelli lemma.

3. (a) Define convergence in quadratic mean and convergence in probability. Prove that $X_n \xrightarrow{a \cdot b} X \Rightarrow X_n \xrightarrow{P} X$.

   (b) State and prove kolmogorov's S.L.L.N. for i.i.d, case.

4. (a) Determine whether weak law of large numbers holds for the following sequence of independent random variables.

$$P(X_n = n) = \frac{1}{2} n^{-\lambda} = p(X_n = -n), P(X_n = 0) = 1 - n^{-\lambda}$$

   (b) State and prove Liapounov form of C.L.T.

5. (a) Derive the compound poisson distribution and find its variance.

   (b) Define negative binomial distribution. Find its m.g.f and hence find its mean and variance.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Statistics

PROBABILITY AND DISTRIBUTION THEORY
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Define the multinomial distribution. Find its moments. Establish the relation between bionomial and multinomial.

   (b) Derive the compound binomial distribution. Give its applications.

2. (a) Suppose $X \sim G(1, \beta)$. Let $Y = X^{1/\alpha}, \alpha > 0$. Derive the distribution of Y and comment.

   (b) Define logistic distribution. Obtain its m.g.f and hence find its mean and variance.

3. (a) Define Laplace distribution. Obtain its m.g.f and hence find its mean and variance.

   (b) Define log-normal distribution. Find its mean and variance. Describe the relation between log - normal and normal distributions.

4. (a) Derive the distribution of T - Statistic.

   (b) Define order statistic. Obtain the joint distribution of j^{th} and k^{th} order statistics. $1 \le j < k \le n$.

5. (a) Derive the distribution of F statistics. Define non central F - Distribution.

   (b) Derive the distribution of range.

(DMSTT02)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Statistics

STATISTICAL INTERFACE

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Define a sufficient statistic. Prove that sample mean and variance are jointly sufficient for μ and σ^2 in $N(\mu, \sigma^2)$ where μ and σ^2 both are unknown.

   (b) State and prove Lehmann – Scheffe's theorem.

2. (a) Let $x_1, x_2, \ldots x_n$ be $N(\theta, 1)$. Show that $\overline{X}^2 - \frac{1}{n}$ is the UMVUE for θ^2.

   (b) State and prove Cramer - Rao inequality.

3. (a) Define CAN estimator. Let $x_1, x_2, \ldots x_n$ be i.i.d with p.d.f. $f(x, \theta) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$. Show that the moments estimator based on $\log x_i$ is CAN for θ.

   (b) Explain the maximum likelihood method of estimation. Prove that ML estimator is asymptotically efficient.

4. (a) Explain consistent and CAUN estimators with example. State and prove a sufficient condition for an estimator to be consistent.

   (b) Explain pivotal quantity method of finding the confidence interval.

5. (a) Distinguish between randomised and non-randomised tests. Give examples. State and prove Neyman-Pearson Lemma.

   (b) Explain likelihood ratio test. Derive its asymptotic distributions.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Statistics

STATISTICAL INTERFACE

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Obtain MP and UMP regions for testing simple hypothesis for the case of random sampling from $N(\mu, \sigma^2)$ when μ known but variance unknown.

   (b) Explain monotone likelihood ratio property. Explain its use in estimation and testing with suitable examples.

2. (a) Show that Kolmogorov - Smirnov statistics are distribution free for any continuous
   d.f. F.

   (b) Define a run. Describe Wold - Wolfowitz two sample run test.

3. (a) Explain Wilcoxon - Mann - Whitney test. Clearly mention the situation when it can be employed and also bring out its special uses.

   (b) Explain sign test for one-sided and two-sided hypothesis. Show that the test is consistent.

4. (a) Explain the SPRT and its OC and ASN functions.

   (b) Explain SPRT for testing $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma = \sigma_1$ in normal case with known mean. Derive its OC and ASN curves.

5. (a) Prove that SPRT eventually terminates with probability one.

   (b) Explain SPRT for testing the proportion of a binomial distribution. Obtain its O.C. and ASN functions.

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DMSTT03

ASSIGNMENT 1

M.Sc. DEGREE EXAMINATION, MAY/JUNE- 2025.

(First Year)

STATISTICS

Sampling Theory

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

Q1) a) Explain sampling and non-sampling errors.

b) What is the difference between enumeration survey and sample survey? Explain the features of sample survey?

Q2) a) What are the important aspects should be considered at planning sample survey?

b) What is a simple random sample? Mention the various methods of drawing a random sample.

Q3) a) Define simple randam process. Explain its merits and drawbacks.

b) Explain fully the concepts of (i) sampling with replacement and (ii) sampling without replacement.

Q4) a) Why stratification is important in sample determination? Explain?

b) How do you determine sample by Neyman allocation method.

Q5) a) How do you determine sample by proportional allocation method.

b) How do you estimate mean and variance with systematic sampling.

DMSTT03

ASSIGNMENT 2

M.Sc. DEGREE EXAMINATION, MAY/JUNE- 2025.

(First Year)

STATISTICS

Sampling Theory

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

Q1) a) Explain cluster sampling with equal cluster sizes.

b) What are the features of cluster sampling?Explain?

Q2) a) What is systematic sampling ? Give illustrations where such sampling is usual.

b) Describe 'circular systematic sampling'.

Q3) a) Write the merits and draw backs of multi-stage sampling.

b) Discuss about the Two-stage sampling with equal number of second stage units.

Q4) a) What are the biases of ratio - estimator ? Explain.

b) Explain the comparison of the ratio estimate with the mean per unit.

Q5) a) What is the differences between ratio estimate and regression estimate.

b) What is the conditions for optimum ratio estimate? Explain.

(DMSTT04)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Statistics

DESIGN OF EXPERIMENTS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Define (i) determinant of a 3×3 matrix (ii) rank of a matrix and (iii) idempotent matrix. Give examples one each. What are the properties of determinants?

   (b) State and prove Cayley-Hamilton theorem.

2. (a) Find the characteristic roots and vectors of the following matrix :

$$\begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$$

   (b) Explain matrix differentiation. Discuss differentiation of $\alpha = X^T AX$ where X is $(n \times 1)$, A is $(n \times n)$ (i) A does not depend on X and (ii) A is a symmetric matrix.

3. (a) Explain the concept of estimable parametric functions. State and prove a necessary and sufficient condition for a linear function to be estimable.

   (b) Explain Gauss-Markov set up. Show that OLS estimations of the model are b.l.u.e.s.

4. (a) Define best linear unbiased estimate of an estimable function. Show that the best estimate of linear function of two estimable functions in the linear function of the best estimates.

   (b) Explain generalised linear model. State and prove Aitken's theorem.

5. (a) Explain fixed, random and mixed effect models. Discuss their applications.

   (b) Explain two-way ANOVA with unequal number of observation per cell and its statistical analysis.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Statistics

DESIGN OF EXPERIMENTS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Explain one-way ANOVA and its statistical analysis.

   (b) Explain two-way analysis of covariance and its statistical analysis.

2. (a) Explain Graeco-Latin square design. Explain the method of estimation and ANOVA of the design.

   (b) Explain RBD with a missing plot and its analysis.

3. (a) Explain the principles of experimental designs. Explain LSD and its statistical analysis.

   (b) Explain CRD and its statistical analysis.

4. (a) Explain 2^3 factorial experiment and its main effects and interactions. Discuss its statistical analysis.

   (b) Explain BIBD. Establish its parametric relations.

5. (a) Explain 3^3 factorial design and its statistical analysis.

   (b) Explain intra block analysis of BIBD.

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