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ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

Mathematics

Second Year

TOPOLOGY AND FUNCTIONAL ANALYSIS

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Show that a subspace of a topological space is itself a topological space.

   (b) Prove that every separable metric space is second countable.

2. (a) State and prove Tychonoff's theorem.

   (b) Prove that every closed and bounded subspace of the real line is compact.

3. (a) State and prove Ascoli's theorem.

   (b) Show that R_∞ is not locally compact.

4. (a) Show that every compact subspace of a Hausdorff space is closed. Deduce that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism.

   (b) Show that a closed subspace of a normal space is normal.

5. (a) State and prove Urysohn's lemma.

   (b) State and prove the Tietze extension theorem.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

Mathematics

Second Year

TOPOLOGY AND FUNCTIONAL ANALYSIS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Define Banach space and give some examples.

   (b) Prove that a normal linear space N is separable if its conjugate space N* is separable.

2. (a) State and prove the uniform boundedness theorem.

   (b) Explain, the open mapping theorem.

3. (a) Explain, the closed Graph theorem.

   (b) State and prove Bessel's Inequality.

4. (a) Define Hilbert space and give some examples.

   (b) Prove that a Hilbert space H is separable if and only if every orthonormal set in H is countable.

5. (a) Show that $\|TT^*\| = \|T\|^2$.

   (b) If H is a closed linear subspace of a Hilbert space H, then show that $H = M \oplus M^{\perp}$.

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ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

Mathematics

Second Year

MEASURE AND INTEGRATION

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Let G be an algebra of subsets and <Ai> a sequence of sets in G. Then show that there is a sequence <Bi> of sets in G such that $B_n \cap B_m = \phi$ for $n \neq m$ and $\bigcup_{i=1}^{\infty} B_i = \bigcup_{i=1}^{\infty} A_i$.

   (b) Explain Hausdorff maximal principle.

2. (a) If f is measurable function and $f = g a.e$, then prove that g is measurable.

   (b) Show that if E_1 and E_2 are measurable then $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$.

3. (a) State and prove Egoroff's theorem.

   (b) Show that every Borel set is measurable. In particular show that each open set and each closed set is measurable.

4. (a) State and prove bounded convergence theorem.

   (b) State and prove Fatou's lemma.

5. (a) Show that if f is integrable over E, then so is $|f|$ and $\left|\int_E f\right| \leq \int_E |f|$.

   (b) Show that if $\langle f_n \rangle$ is a sequence that converges to f is measure then each subsequence $\langle f_n \rangle$ converges to f in measure.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

Mathematics

Second Year

MEASURE AND INTEGRATION
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) If f is absolutely continuous on [a b] and $f'(x) = 0$ a.e, then prove that f is constant.

   (b) Let f be defined by $f(o) = 0$ and $f(n) = x^2 \sin\left(\frac{1}{x^2}\right)$ for $x \neq 0$. If f is bounded variation on [–1, 1].

2. (a) State and prove the Minkowski in equality.

   (b) Prove that if $f \in L^1$ and $g \in L^\infty$ then $\int |fg| \leq \|f\|_1 \|g\|_\infty$.

3. (a) State and prove the Radon-Nikodym theorem.

   (b) Let E be a measurable set such that $0 < vE < \infty$. Then show that there is a positive set A contained in E with $A > 0$.

4. (a) State and prove Hahn decomposition theorem.

   (b) Show that if v_1 and v_2 are singular with respect to μ then so is $c_1 v_1 + c_2 v_2$.

5. (a) If $A \in G$, then show that A is measurable with respect to μ^*.

   (b) If μ^* is a caratheodory outer measure with respect to Γ, then prove that every function in Γ is μ^*–measurable.

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ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

Second Year

Mathematics

ANALYTICAL NUMBER THEORY AND GRAPH THEORY
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) State and prove Euler's Summation formula.

   (b) Show that for all $x \geq 1$,

   $$\sum_{n \leq x} \sigma_1(n) = \frac{1}{2}\ \xi\ (2) + O(x \log x).$$

2. (a) For all $x \geq 1$, prove that $\left| \sum_{n \leq x} \frac{\mu(n)}{n} \right| \leq 1$ with equality holding only if $x < 2$.

   (b) State and prove Legender's identity.

3. (a) For every integer $n \geq 2$, prove that $\frac{1}{6} \frac{n}{\log n} < \pi(n) < 6 \frac{n}{\log n}$.

   (b) For $x > 0$, prove that $O \leq \frac{\psi(x)}{x} - \frac{\theta(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x}.\log 2}$.

4. (a) State and prove Selberg's asymptotic formula.

   (b) Prove that the prime number theorem implies $\lim_{n \to \infty} \frac{M(n)}{x} = 0$.

5. (a) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

   (b) Prove that any two simple connected graphs with n vertices, all of degree '2' is isomorphic.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

Second Year

Mathematics

ANALYTICAL NUMBER THEORY AND GRAPH THEORY
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Explain Hamilton graph with an example.

   (b) Prove that, an Euler graph G is arbitrary traceable from vertex V in G if and only if every circuit in G contains V.

2. (a) Prove that there is one and only one path between every pair of vertices in a tree T.

   (b) Prove that every connected graph has at least one spinning tree.

3. (a) Prove that a graph is a tree if and only if it is minimally connected.

   (b) Prove that in every non trivial tree there is atleast two vertex whose degree is one.

4. (a) Prove that complete graph of five vertices is non planar.

   (b) Prove that Kuratowski's is second graph is non-planar.

5. (a) Explain about Modular arithmetic and Galois fields.

   (b) Define terms

   (i) Commutative ring with Identity

   (ii) Division ring

   (iii) Vector space

   (iv) Field and give an example of each

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ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

Second Year

Mathematics

RINGS AND MODULES
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Define a Boolean ring.

   In a Boolean ring B prove that

   (i) $a+a=0$ for all $a \in B$

   (ii) $ab-ba$ for all $a, b \in B$.

   (b) Let R and S be two rings and $\phi: R \to S$ be a ring homomorphism. Then prove that, for any ideal J of S, $\phi^{-1}(J)$ is an ideal of R.

2. (a) Prove that the union of simply ordered family of subrings is a subring.

   (b) Define a maximal ideal of a ring and prove that every proper ideal of a ring is contained a maximal proper ideal.

3. (a) If B and C are submodules of A then prove that $B+C/B$ is isomorphic to $C/B \cap C$.

   (b) Verify that $Hom_R(A,B)$ is an Abelian group.

4. (a) If C is a central idempotent in a ring R then prove that eR is indecomposable if and only if e is an atom of B(R).

   (b) If R is a direct sum of indecomposable ideals then prove that these are the only indecomposable direct summands of R.

5. (a) If R is a Boolean ring then prove that Q(R) is a Boolean ring.

   (b) Prove that a commutative ring is a field if and only if e is a maximal ideal.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

Second Year

Mathematics

RINGS AND MODULES
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Prove that the ring R is primitive if and only if there exists a faithful irreducible module A_R.

   (b) If K and P are ideals such that $K \subset P \subset R$, show that P/K is prime if and only if P is prime.

2. (a) Show that, the prime radical of R is the set of all strongly nilpotent elements.

   (b) Prove that, the radical of R is the set of all rR such that 1-rS is right invertible for all $S \in R$.

3. (a) Prove the following conditions concerning the module A are equivalent

      (i) A is completely reducible

      (ii) A has no proper large sub-module.

      (iii) L(A) is complemented.

   (b) Show that a right artinian ring without zero - divisors is a division ring.

4. (a) Prove that every free module is projective.

   (b) Prove that every module is isomorphic to a factor of a free modules.

5. (a) Prove that M is injective if and only if M has no proper essential extension.

   (b) Prove that every module is isomorphic to a submodule of an injective module.