(DM01)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Mathematics

ALGEBRA

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) State and prove the Cauchy's theorem for abelian groups.

   (b) If ϕ is a homomorphism of a group G into a group $\overline{G}$ with kernel K, then prove that K is a normal subgroup of G.

2. (a) State and prove Cayley's theorem.

   (b) If P is a prime number and $P^{\alpha} / O(G)$, then prove that G has a subgroup of order P^{α}.

3. (a) State and prove the Sylow's theorem.

   (b) Compute $a^{-1}ba$, where $\begin{matrix} a = (1,3,5)(1,2) \\ b = (1,5,7,9) \end{matrix}$

4. (a) State and prove the division algorithm.

   (b) Prove that $j[i]$ is a Euclidean ring.

5. (a) State and prove the Fermat's theorem.

   (b) Prove that if K is any field which contains D then K contains a sub field isomorphic to F.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Mathematics

ALGEBRA

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Prove that the number e is transcendental.

   (b) Prove that the mapping $\psi : F[x] \to F(a)$ defined by $h(n)\psi = h(a)$ is a homomorphism.

2. (a) Prove that the group G is solvable if and only if $G^{(K)} = (e)$ for some integer K, with the usual notation.

   (b) Show that a polynomial of degree n over a field can have atmost n roots in any extension field.

3. (a) Show that a general polynomial of degree $n, n \geq 5$ is not solvable by radicals.

   (b) Let k is a normal extension of F if and only if k is splitting field of some polynomial over F.

4. (a) State and prove Schreier's theorem.

   (b) Show that the lattice of subgroups of A_4 is not modular.

5. (a) Prove that every distributive lattice with more than one element can be represented as a sub direct union of two element chains.

   (b) Define Boolean algebra and Boolean ring. Show that a Boolean ring can be converted into a Boolean algebra.

(DM02)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Mathematics

ANALYSIS

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) Prove that a set E is open if and only if complement is closed.

   (b) Prove that every K-cell is compact.

2. (a) Prove that closed subsets of compact sets are compact.

   (b) Suppose $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

3. (a) Show that the product of two convergent series need not converge and may actually leverage.

   (b) Prove that if $P > 1$, the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^P}$ converge and if $P \le 1$, the series is diverges.

4. (a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that $f(X)$ is compact.

   (b) Let f be monotonically increasing on (a, b). Then prove that $f(x_+)$ and $f(x_-)$ exist for all $x \in (a, b)$ and if $a < x < y < b$ then $f(x_+) \le f(y_-)$.

5. (a) Prove that, if f is continuously on $[a, b]$, then $f \in \mathbb{R}(\alpha)$ on $[a, b]$.

   (b) State and prove a necessary and sufficient condition for a bounded function f to be $R-S$ integrable on $[a, b]$.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Mathematics

ANALYSIS

MAXIMUM MARKS :30

ANSWER ALL QUESTIONS

1. (a) State and prove the fundamental theorem of integral calculus.

   (b) If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$ then prove that $f \in \mathbb{R}(\alpha)$.

2. (a) State and prove Weirstrass approximation theorem.

   (b) Prove that every uniformly convergent sequence of bounded function is uniformly bounded.

3. (a) Show that $\sum r^n \sin n\theta$ and $\sum r^n \cos n\theta$ converge uniformly for all values of θ, if $0 < r < 1$.

   (b) Let $\mathbb{R}$ be the uniform closure of an algebra of bounded functions. Then $\mathbb{R}$ is a uniformly closed Algebra.

4. (a) State and prove the Lebesgue's dominated convergence theorem.

   (b) Show that the continuous functions form a dense subset of Z^2 on $[a, b]$.

5. (a) State and prove the Riesz-Fischer theorem.

   (b) If $\{f_n\}$ is a sequence of measurable functions, prove that the set of points x at which $\{f_n(x)\}$ converges is measurable.

(DM03)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Mathematics

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS AND PARTIAL DIFFERENTIAL EQUATIONS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Prove that
$$\int_{-1}^{1} p_m(x)p_n(x)dx = \begin{cases} 0 & if \quad m \neq n \\ \dfrac{2}{2n+1} & if \quad m = n \end{cases}.$$

   (b) Show that $\int_{0}^{1} p_{2n}(x)dx = 0$.

2. (a) Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendres polynomial.

   (b) Find the solution of the Bessels equation of order n and if the first kind , n being a non negative constant.

3. (a) Show that $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(n)$.

   (b) Prove that $J_3(x) = \left[\frac{8}{x^2} - 1\right] J_1(x) - \frac{y}{x}J_0(x)$.

4. (a) Solve $(D^2 - 2DD' + D^2)z = e^{x+2y}$.

   (b) Solve $y^3 r - 2ys + t = p + 6y$ using Monge's method.

5. (a) Solve the partial differential equation $(x^2 + y^2 + yz)p + (x^2 + y^2 - xz)q = z(x + y)$, with the usual notation.

   (b) Find the general solution of the partial differential equation $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^x$.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Mathematics

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS AND PARTIAL DIFFERENTIAL EQUATIONS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} z^{n!}$.

   (b) If $f(z)$ is analytic function with constant modulus, show that $f(z)$ is constant.

2. (a) State and prove Cauchy Integral formula.

   (b) State and prove open mapping theorem.

3. (a) State and prove Goursatils theorem.

   (b) What type of singularity have the function $f(z) = \frac{e^{2z}}{(z-1)^4}$.

4. (a) By integrating around DC unit circle, evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$.

   (b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{a+b}$ $(a, b > 0)$.

5. (a) State and prove the Residue theorem.

   (b) State and prove the Liouville's theorem.

(DM04)

ASSIGNMENT-1

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Mathematics

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) If one solution of $y'' - \frac{2}{x^2} y = 0$, $0 < x < \infty$ is $\phi_1(x) = x^2$, find the general solution of the equation $y'' - \frac{2}{x^2} y = x$.

   (b) Let x_0 in I and $\alpha_1, \alpha_2, ... \alpha_n$ be constants. Prove that there is atmost one solution ϕ of $L(y) = 0$ on I satisfying $\phi(x_0) = \alpha_1$, $\phi'(x_0) = \alpha_2 ...$, $\phi^{(n-1)}(x_0) = \alpha_n$.

2. (a) Find two linearly independent power series solutions of the equation $y'' - xy = 0$.

   (b) Compute the solution of $y''' - xy = 0$ which satisfies $\phi(0) = 1$, $\phi'(0) = 0$, $\phi''(0) = 0$.

3. (a) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y' = 1 + xy$, $y(0) = 1$.

   (b) Find all real value solution of $y' = \frac{x + x^2}{y - y^2}$.

4. (a) State and prove the existence theorem for the convergence of successive approximations to a solution ϕ of $y' = f(x, y)$, $y(x_0) = y_0$.

   (b) Let $f(x, y) = \frac{\cos y}{1 - x^2} (|x| < 1)$ then show that f satisfies a Lipschitz condition and every strip $sa : |x| \le a$ where $0 < a < 1$.

5. (a) Find a solution ϕ of the system $y' = y_2$, $y_2' = 6y_1 + y_2$, satisfying $\phi(0) = (1, 1)$.

   (b) Solve the equation $y'' + e^x y' = e^x$.

ASSIGNMENT-2

M.Sc. DEGREE EXAMINATION, MAY/JUNE -2025

First Year

Mathematics

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS
MAXIMUM MARKS :30
ANSWER ALL QUESTIONS

1. (a) Show that all real valued solutions of the equation $y'' + \sin y = b(x)$, where b is continuous for $-\infty < x < \infty$, exist for all real x.

   (b) Find a solution ϕ of the system $y_1' = y_1, y_2' = y_1 + y_2$ which satisfies $\phi(0) = (1, 2)$.

2. (a) Find the general solution of the Riccate equation.

   (b) Find the functions $z(x), k(x)$ and $m(x)$ such that $z(x).\ \left[x^2 y'' - 2xy' + 2y\right] = \dfrac{d}{dx}\left[k(x)y' + m(x)y\right]$.

3. (a) Show that the Green's function for $L(x) = x'' = 0$, $x(0) + x(1) = 0$, $x'(0) + x'(1) = 0$ is $G(t, s) = 1 - s$ if $t \le s$ and $G(t, s) = 1 - t$ if $t \ge s$.

   (b) Find the general solution of $y'' - 4y' + 3y = x$, $(-\infty < x < 0)$ by computing the particular solution using Green's theorem.

4. State and prove the sturm separation theorem.

5. (a) State and prove the Bocher OS good theorem.

   (b) State and prove Abel's formula.