(DMSTT01)

ASSIGNMENT-1 M.Sc. DEGREE EXAMINATION, DEC. – 2017 First Year STATISTICS Probability and Distribution Theory

MAXIMUM MARKS:30

- **Q1)** a) Explain (i) statistical independence and (ii) convolution of random variables. Give examples.
 - b) Define distribution. State and prove its properties.
- **Q2)** a) Give the axiomatic definition of probability. State its properties. State and prove Kolmogorov in equality.
 - b) Define characteristic function. State its properties. State and prove inversion theorem.
- **Q3)** a) State and prove Chebyshev's WLLN and Khintchine's WLLN.
 - b) Examine whether Kolmogorov's SLLN holds for the sequence of *i.i.d* random variables with $p.d.f: f(x) = \frac{e}{x^2 \log x}, x > e$.
- Q4) a) Explain the types of convergence. Show that convergence in distribution is weaker than convergence in probability and hence convergence a.s.
 - b) State and prove Lindberg-Levy form of central limit theorem.
- Q5) a) Define Hypergeometric distribution. Obtain its *m.g.f* and hence its mean and variance.
 - b) Derive compound binomial distribution.

(DMSTT01)

ASSIGNMENT-2 M.Sc. DEGREE EXAMINATION, DEC. – 2017 First Year STATISTICS Probability and Distribution Theory

MAXIMUM MARKS:30

- _*Q1*) a) State and prove the reproductive property of Poisson random variables.
 - b) Derive compound Poisson distribution.
- **Q2)** a) Define log-normal distribution. State and prove its reproductive property.
 - b) Define logistic distribution. Obtain its *m.g.f.*
- **Q3)** a) Define Weibull distribution. Obtain its characteristic function. Hence obtain its mean and variance.
 - b) Define Laplace distribution. Obtain its characteristic function and hence its mean and variance.
- Q4) a) Derive the *p.d.f* of t-statistic.
 - b) Obtain the joint *p.d.f* of j^{th} and k^{th} order statistics. Find the marginal *p.d.f* of r^{th} order statistics.
- **Q5)** a) Derive the *p.d.f* of non-central Chi-square.
 - b) Derive the *p.d.f* of F-statistics with (m,n) degrees of freedom.



(DMSTT02)

ASSIGNMENT-1 M.Sc. DEGREE EXAMINATION, DEC. – 2017 First Year STATISTICS Statistical Inference

MAXIMUM MARKS:30

- **Q1)** a) State and prove Rao-Blackwell theorem.
 - b) State and prove the uniqueness property of the MVU estimator.
- (Q2) a) Derive the Cramer-Rao lower bound for the parameter θ in $f(x:\theta) = \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right), x \ge 0, \theta > 0.$
 - b) State and prove Lehmann-Scheffe theorem.
- **Q3)** a) Explain *m.l* method of estimation. State the properties of *m.l.* estimators. Find *m.l.* estimators of θ_1 and θ_2 in $f(x:\theta_1,\theta_2) = \frac{1}{\theta_1} \exp(-(x-\theta_2)/\theta_1), \theta_2 \le x < \infty, \theta_1 > 0$ and $-\infty < \theta_2 < \infty$.
 - b) Define CAN and CAUN estimators. Describe the construction of CAN estimators based on percentiles.
- Q4) a) Obtain confidence limits for the parameter μ in N(μ , 1) with confidence coefficient $(1-\alpha)$.
 - b) Describe *m.l.* method of estimation. Obtain *m.l.* estimators of μ and σ^2 based on a random sample of size *n* from N(μ , σ^2).
- **Q5)** a) State and prove Neyman-Pearson Lemma.
 - b) Consider *n* Bernoullian trials with probability of success *p* for each trial. Derive the likelihood ratio test for $H_0 : p = p_0$ against $H_1 : p > p_0$ and $H_1 : p < p_0$. Show that they are identical with the corresponding UMP tests.

(DMSTT02)

ASSIGNMENT-2 M.Sc. DEGREE EXAMINATION, DEC. – 2017 First Year STATISTICS Statistical Inference

MAXIMUM MARKS:30

- **_Q1)** a) Explain likelihood ratio test procedure. Obtain the asymptotic distribution likelihood ratio test.
 - b) Obtain the likelihood ratio test for testing H_0 : $\theta = \theta_0$ against the alternative H_1 : $\theta > \theta_0$ in the case of N(θ , σ^2), σ^2 is known. Show that it is identical with the corresponding UMP test.
- **Q2)** a) Explain Wilcoxon signed rank test.
 - b) Explain in Kolmogorov-Smirnov one sample and two sample tests.
- **Q3)** a) Explain Wilcoxon Mann-Whitney test.
 - b) Explain sign test.
- Q4) a) Describe SPR test and show that it terminates with probability one.
 - b) Derive SPR test to test the parameter λ of a Poisson distribution. Obtain its OC and ASN functions.
- **Q5)** a) Determine the constants A and B in SPR test.
 - b) Derive SPR test to test the mean of N(μ , σ^2), σ^2 is known. Obtain OC and ASN functions.



(DMSTT03)

ASSIGNMENT-1 M.Sc. DEGREE EXAMINATION, DEC. – 2017 First Year STATISTICS Sampling Theory

MAXIMUM MARKS:30

- **Q1)** a) What is a sample survey? In what respect is it superior to a census survey? Distinguish between sampling and non-sampling errors.
 - b) Discuss the main steps involved in a sample survey.
- **Q2)** a) Discuss the basic principles of a sample survey.
 - b) Explain the organization and functions of NSSO.
- Q3) a) Derive the formula for the sample size in the case of continuous data.
 - b) Explain the allocation problems in stratified random sampling.
- **Q4)** a) Explain the purpose of stratification in sample surveys. Under what conditions is stratified random sampling preferred to simple random sampling and why?
 - b) Compare simple random sampling and stratified random sampling with proportional and optimum allocations. When does stratification produce large gains in precisions.
- **Q5)** a) What is cluster sampling? Give its applications. Obtain the variance of the estimated mean in cluster sampling with clusters of equal sizes.
 - b) Explain systematic sampling. What are its advantages and disadvantages? Obtain the relative efficiency of systematic sampling as compared to SRSWOR.

(DMSTT03)

ASSIGNMENT-2 M.Sc. DEGREE EXAMINATION, DEC. – 2017 First Year STATISTICS Sampling Theory

MAXIMUM MARKS:30

- **_Q1**) a) Determine the optimum cluster size so as to minimize the variance for a fixed cost.
 - b) Explain circular systematic sampling. Derive the variance of the estimated mean in systematic sampling.
- **Q2)** a) Explain Lahiri's method of PPS sampling. Derive the variance of the estimated population total in PPS sampling with replacement.
 - b) Explain multistage sampling. What are its advantages and disadvantages? Give its applications.
- **Q3)** a) Obtain an unbiased estimate of the population total in PPS sampling with replacement. Derive the variance of the estimated population total.
 - b) Explain two-stage sampling. What are its applications? Obtain the estimate of variance of the estimated mean in two-stage sampling with equal second stage units.
- **Q4)** a) Derive the conditions under which ratio estimator is BLUE. Compare the ratio estimate with the mean per unit.
 - b) Obtain the variance of the linear regression estimate with preassigned regression coefficient (b_0) . Determine the best choice for b_0 .
- **Q5)** a) Define ratio estimates in stratified random sampling and obtain $V(\hat{R}_{rs})$ in the usual notation.
 - b) Discuss the relative efficiencies of ratio and regression estimates.



(DMSTT04)

ASSIGNMENT-1 M.Sc. DEGREE EXAMINATION, DEC. – 2017 (First Year) STATISTICS Design of Experiments MAXIMUM MARKS:30

Answer ALL Questions

- **Q1)** a) Explain (i) differentiation using matrices (ii) trace of a matrix and (iii) idempotent matrix with examples.
 - b) Find the characteristic roots and vectors of $A = \begin{pmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ 2 & 2 & -3 \end{pmatrix}$.

Q2) a) State and prove Cauley-Hamilton theorem.

b) State Cochran's theorem for quadratic forms.

Find the determinant of A =
$$\begin{pmatrix} 1 & 2 & 5 & 2 \\ -2 & 3 & 0 & -4 \\ 1 & -1 & 0 & 2 \\ 0 & 1 & 4 & 2 \end{pmatrix}.$$

- **Q3)** a) Explain the linear model. Define best linear unbiased estimate.
 - b) State and prove Gauss-Markov theorem.
- **Q4)** a) Explain the generalized linear model. State and prove a necessary and sufficient condition for the estimability of a linear parametric function.
 - b) State and prove Aitken's theorem.
- **Q5)** a) Explain analysis of variance of one-way classification with equal number of observations.
 - b) Explain analysis of covariance of two-way classification.

(DMSTT04)

ASSIGNMENT-2 M.Sc. DEGREE EXAMINATION, DEC. – 2017 (First Year) STATISTICS Design of Experiments MAXIMUM MARKS:30

- **_Q1)** a) Explain analysis of variance of two-way classification with unequal number of observations.
 - b) Explain analysis of covariance of one-way classification.
- **Q2)** a) Explain the analysis of RBD.
 - b) Explain Gracco Latin Square design. Give its analysis.
- *Q3)* a) What are mutually orthogonal LSDs? How do you construct them?b) Explain the analysis of LSD with a missing row or a missing column.
- Q4) a) What are factorial experiments? Explain the analysis of 3^3 factorial experiment.
 - b) Define BIBD. State and prove its parametric relations.
- **Q5)** a) Explain the analysis of 2^2 factorial experiment.
 - b) Discuss the intra block analysis of BIBD.

