

ASSIGNMENT - 1, DEC - 2016.

M.Sc. (PREVIOUS) FIRST YEAR DEGREE

MATHEMATICS

PAPER- I : ALGEBRA

Maximum : 30 MARKS

Answer ALL questions.

- 1) a) Let ϕ be a homomorphism of G onto \bar{G} with Kernel. Then prove that $G/K \cong \bar{G}$.
b) State and prove Cauchy's theorem for abelian groups.
 - 2) a) If $O(G) = P^n$, where P is prime number, then show that $Z(G) \neq (e)$.
b) Prove that the number of conjugate class in S_n is $P(n)$, the number of partitions of n .
 - 3) a) State and prove second part of Sylow's theorem.
b) Prove that the number of nonisomorphic abelian groups of order P^n , P a prime, equals the number of partitions of n .
 - 4) a) Prove that a finite integral domain is a field.
b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is field.
 - 5) a) State and prove the Eisenstein criterion theorem.
b) If $a \in R$ is an irreducible element and $a|bc$, then show that $a|b$ or $a|c$.
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ASSIGNMENT - 2, DEC - 2016.

M.Sc. (PREVIOUS) FIRST YEAR DEGREE

MATHEMATICS

PAPER- I : ALGEBRA

Maximum : 30 MARKS

Answer ALL questions.

- 1) a) If L is a finite extension of K and if K is a finite extension of F , then show that L is a finite extension of F . Moreover $[L:F] = [L:K][K:F]$.
b) Prove that the number e is transcendental.
 - 2) a) Prove that S_n is not solvable for $n \geq 5$.
b) If $P(x) \in F(x)$ is solvable by radicals over F , then prove that the Galois group over F of $P(x)$ is a solvable group.
 - 3) State and Prove the fundamental theorem of Galois theory.
 - 4) State and prove Schreier's theorem.
 - 5) Prove that every distributive lattice which more than one element can be represented as a subdirect union of two element chains.
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M.Sc. (PREVIOUS) FIRST YEAR DEGREE

MATHEMATICS

PAPER- II : ANALYSIS

Maximum : 30 MARKS

Answer ALL questions.

- 1) a) Prove that every infinite subset of a countable set A is countable.
b) Prove that compact subsets of metric spaces are closed.
 - 2) a) Prove that every k -cell is compact.
b) Let P be a nonempty perfect set in \mathbb{R}^k . Then show that P is uncountable.
 - 3) a) Show that the product of two convergent series need not converge and may actually diverge.
b) Suppose $\{S_n\}$ is monotonic. Then show that $\{S_n\}$ converges if and only if it is bounded.
 - 4) a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y . Then show that $f(X)$ is compact.
b) Let f be a continuous mapping of a compact metric space X into a metric space Y . Then show that f is uniformly continuous on X .
 - 5) a) Let f be monotonic on (a, b) . Then show that the set of points of (a, b) at which f is discontinuous is at most countable.
b) If f is continuous on $[a, b]$, then show that $f \in R(\alpha)$ on $[a, b]$.
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ASSIGNMENT - 2, DEC - 2016.

M.Sc. (PREVIOUS) FIRST YEAR DEGREE

MATHEMATICS

PAPER- II : ANALYSIS

Maximum : 30 MARKS

Answer ALL questions.

- 1) a) Suppose $f \in R(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$, and $h(x) = \phi(f(x))$ on $[a, b]$. Then show that $h \in R(\alpha)$ on $[a, b]$.
- b) Assume α increases monotonically and $\alpha' \in R$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then show that $f \in R(\alpha)$ if and only if $f \alpha' \in R$. In that case
- $$\int_a^b f d\alpha = \int_a^b f(x) \alpha'(x) dx.$$
- 2) a) Prove that the sequence of functions $\{f_n\}$, defined on E , converges uniformly on E if and only if for every $\varepsilon > 0$ there exists an integer N such that $m \geq N, n \geq N, x \in E$ implies $|f_n(x) - f_m(x)| \leq \varepsilon$.
- b) Prove that there exists a real continuous function on the real line which is not differentiable.
- 3) State and Prove Weierstrass approximation theorem.
- 4) a) Let f and g be measurable real valued functions defined on X , let F be real and continuous on R^2 , and put $h(x) = F(f(x), g(x)), (x \in X)$.
Then show that h is measurable. In particular, $f + g$ and fg are measurable.
- b) State and prove Fatou's theorem.
- 5) a) If $f \in \mathcal{L}(\mu)$ on E , then show that $|f| \in \mathcal{L}(\mu)$ on E and $\left| \int_E f d\mu \right| \leq \int_E |f| d\mu$.
- b) State and prove Lebesgue's dominated convergence theorem.
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M.Sc. (PREVIOUS) FIRST YEAR DEGREE

MATHEMATICS

PAPER- III : COMPLEX ANALYSIS AND SPECIAL FUNCTIONS AND PARTIAL
DIFFERENTIAL EQUATIONS

Maximum : 30 MARKS

Answer ALL questions.

1) a) Prove that $(2n+1)x p_n(x) = (n+1) p_{n+1}(x) + n p_{n-1}(x)$.

b) Prove that $J_{5/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \left\{ \frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right\}$

2) a) Prove that $\int_{-1}^1 x p_n p_m' dx$ either 0 or 2 or $\frac{2n}{2n+1}$.

b) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.

3) a) Evaluate $\int x^3 J_3(x) dx$,

b) Derive the Rodrigue's formula.

4) a) Solve $(r + s - 6t) = y \cos x$.

b) Solve $(yz + 2x) dx + (zx - 2z) dy + (xy - 2y) dz = 0$.

5) a) Solve $py + qx = xyz^2(x^2 - y^2)$.

b) Find the complete integral of the equation $2(z + xp + yq) = yp^2$.

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MATHEMATICS

PAPER- III : COMPLEX ANALYSIS AND SPECIAL FUNCTIONS AND PARTIAL
DIFFERENTIAL EQUATIONS

Maximum : 30 MARKS

Answer ALL questions.

1) a) If $f(z)$ is an analytic function, Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

b) Find the radius of convergence of $\sum_{n=1}^{\infty} n^n z^n, z \in c$.

2) a) i) State and prove the symmetry principle.

ii) Discuss the mapping properties of $\cos z$ and $\sin z$.

b) If r is a piecewise smooth and $f : [a, b] \rightarrow c$ is continuous then prove that

$$\int_a^b f dr = \int_a^b f(t)r'(t)dt$$

3) a) State and prove Liouville's theorem. Deduce the fundamental theorem of algebra.

b) State and prove Cauchy's Integral formula.

4) a) State and prove Schwarz's lemma.

b) Prove that $\int_0^{2\pi} \frac{d\theta}{2 - \sin \theta} = \frac{2\pi}{\sqrt{3}}$

5) a) Show that $\int_0^{\infty} \frac{x^{-a}}{1+x} dx = \frac{\pi}{\sin \pi a}$ if $0 < a < 1$ using residue theory.

b) State and prove Rouché's theorem.

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M.Sc. (PREVIOUS) FIRST YEAR DEGREE

MATHEMATICS

PAPER- IV : THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

Maximum : 30 MARKS

Answer ALL questions.

- 1) a) Show that there exists n linearly independent solutions of $L(y)=0$. On I .
- b) If $\phi_1, \phi_2, \dots, \phi_n$ are n solutions of $L(y)=0$ on an interval I , show that they are linearly independent there if and only if, $w(\phi_1, \phi_2, \dots, \phi_n)(x) \neq 0$ for all x in I .
- 2) a) Find all solutions of $x^2 y'' - 4xy' + (2 + x^2)y = x^2$ for $x > 0$.
- b) Find two linearly independent solutions of $y'' - 2xy' + 2\alpha y = 0$, where α is constant.
- 3) a) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y' = 1 + xy, y(0) = 1$.
- b) Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle $R : |x - x_0| \leq a, |y - y_0| \leq b$. Then show that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if, and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R .
- 4) a) Show that the function f given by $f(x, y) = y^{1/2}$ does not satisfy a Lipschitz condition on $R : |x| \leq 1, 0 \leq y \leq 1$.
- b) Find the integrating factor and solve the equation $(e^y + xe^y)dx + xe^y dy = 0$.
- 5) a) Find the solution ϕ of $y'' = 1 + (y')^2$ which satisfies $\phi(0) = 0, \phi'(0) = -1$.
- b) State and prove Local existence theorem.
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ASSIGNMENT - 2, DEC - 2016.

M.Sc. (PREVIOUS) FIRST YEAR DEGREE

MATHEMATICS

PAPER- IV : THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

Maximum : 30 MARKS

Answer ALL questions.

- 1) a) Find a solution ϕ of the system $y'_1 = y_1, y'_2 = y_1 + y_2$ which satisfies $\phi(0) = (1, 2)$.
- b) Consider the system $y'_1 = 3y_1 + xy_3, y'_2 = y_2 + x^3y_3, y'_3 = 2xy_1 - y_2 + e^x y_3$. Show that every initial value problem for this system has a unique solution which exists for all real x .
- 2) a) Find a function $z(x)$ such that $z(x)[y'' + y] = \frac{d}{dx}[k(x)y'm(x)y]$.
- b) Derive an adjoint equation for $Ly \equiv y' - Ay = 0$, where A is $n \times n$ matrix and obtain a condition for the operator L to be self adjoint.
- 3) a) Study the solutions of the Riccati equation $z' + z - e^z, z^2 - e^{-z} = 0$.
- b) Construct Green's function for the problem $u'' = 0, u(0) = 0, u(1) = 0$.
- 4) a) State and prove Sturm- Picone theorem.
- b) Prove that if $\int_1^\infty \left[xp(x) - \frac{1}{4x} \right] dx = +\infty$, the solutions of $y'' + p(x)y = 0$ are oscillatory on $(1, \infty)$.
- 5) a) State and prove Sturm comparison theorem.
- b) Prove that between every pair of consecutive zeros of $\sin x$ there is one zero of $\sin x + \cos x$.
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